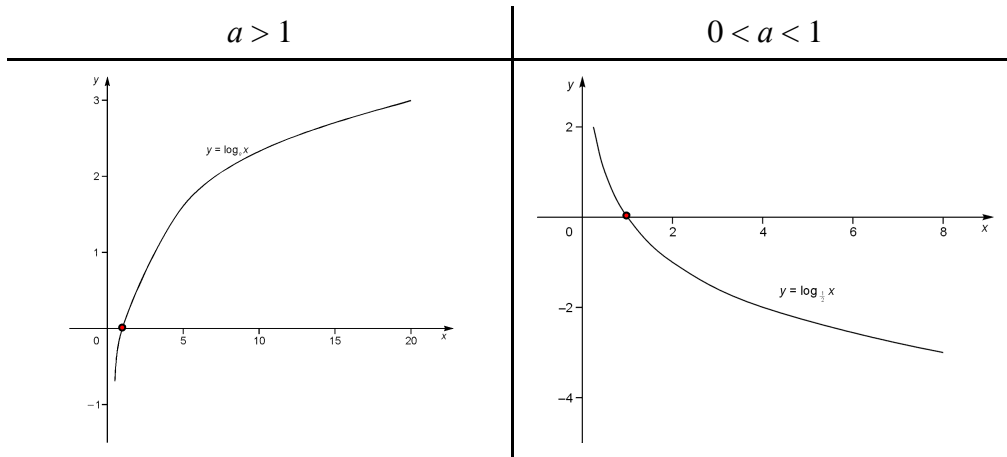


### I. Logarithmic Functions

#### A. Definition

$y = \log_a x$  if and only if  $x = a^y$  where  $a$  is base,  $x > 0$ ,  $a > 0$  and  $a \neq 1$ .

#### B. Graphs



#### Properties of Logarithmic Functions

- a.  $\log_a x$  is defined only for  $a > 0$ ,  $a \neq 1$  and  $x > 0$ .
- b. When  $x$  increases,  $\log_a x$  increases if  $a > 1$  but decreases if  $0 < a < 1$ .
- c. For  $a > 1$ ,
 
$$\log_a x \begin{cases} > 0 & \text{for } x > a \\ = 0 & \text{for } x = a \\ < 0 & \text{for } x < a \end{cases}$$
- d.  $\log_a a = 1$

#### C. Special Cases

	Common log	Natural log
	$a = 10$	$a = e$
$\log_a x$	$\log x$	$\ln x$

### II. Laws of Logarithms (Certificate Level)

- a.  $\log_a 1 = 0$
- b.  $\log_a a = 1$
- c.  $a^{\log_a x} = x$
- d.  $\log_a (xy) = \log_a x + \log_a y$
- e.  $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$
- f.  $\log_a x^n = n \log_a x$
- g. Change of bases  
 $\log_b x = \frac{\log_a x}{\log_a b}$  or  $\log_b x = \frac{\log x}{\log b}$  where  $a$  is assumed to be 10.

### III. Logarithmic Transformation

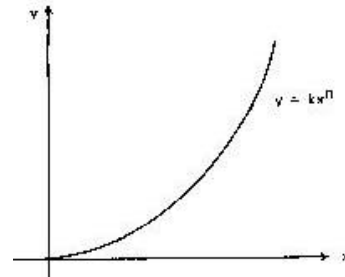
Find the values of  $k$  and  $n$  for the equation  $y = kx^n$ ?

#### A. Procedures

If  $y = kx^n$ , then  $\log_{10} y = n \log_{10} x + \log_{10} k$  or  $Y = nX + C$   
 Then equation  $Y = nX + C$  represents a straight line on the X-Y coordinate system with **slope  $n$**  and  **$y$ -intercept  $C$** .

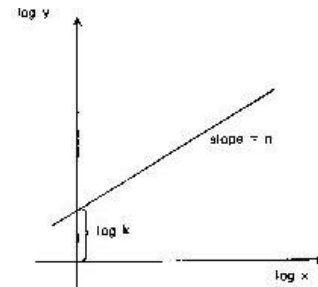
#### B. Theory

If  $y = kx^n$ , the graph of  $y$  against  $x$  is a curve. From the graph,  $k$  and  $n$  cannot be determined.



If logarithm is taken, i.e.  $\log y = n \log x + \log k$

then the graph of  $\log y$  against  $\log x$  is a straight line. From the graph,  $n$  is slope and  $\log k$  is  $y$ -intercept. Thus  $n$  and  $k$  can be found.



#### Example

Corresponding values of the observed quantities  $x$  and  $y$  are given in the following table :-

$x$	50	300	600	900	1200
$y$	0.707	10.4	29.4	54.0	83.1

By plotting  $\log y$  against  $\log x$ , show that they satisfy the relation  $y = kx^n$ . Hence find the values of  $k$  and  $n$ .

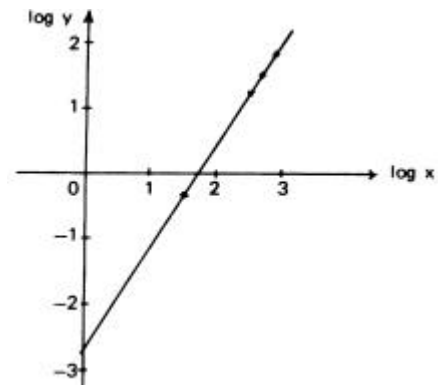
#### Solution

$\log x$	1.699	2.477	2.778	2.954	3.079
$\log y$	-0.151	1.017	1.468	1.732	1.920

From the graph,

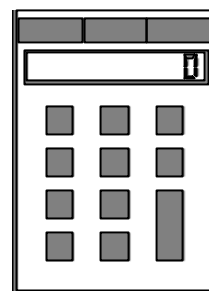
$$\begin{aligned} y\text{-intercept} &= -2.7 \\ \log k &= -2.7 \\ k &= 0.002 \\ \text{slope} = n &= \frac{2.7}{1.8} \\ n &= 1.5 \end{aligned}$$

Hence,  $y = 0.002 x^{1.5}$



IV. Use Calculator(LR mode) to handle Logarithmic transformation (CASIO)

x	1	1.5	2.0	2.5	3.0
y	0.0132	0.0151	0.0167	0.0177	0.0211



STEPS			
MODE	LR	KAC	
1	xD, yD	0.0132	DATA
1.5	xD, yD	0.0151	DATA
2	xD, yD	0.0167	DATA
2.5	xD, yD	0.0177	DATA
3	xD, yD	0.0211	DATA

After Key in :

1. Ans
- |   |                         |                       |
|---|-------------------------|-----------------------|
| r | correlation coefficient | 0.9814                |
| A | y - intercept           | $9.4 \times 10^{-3}$  |
| B | slope                   | $3.68 \times 10^{-3}$ |

2. Estimates of other values on the line :

- Ans
- |       |           |             |        |
|-------|-----------|-------------|--------|
| 2.2   | $\hat{y}$ | estimated y | 0.0175 |
| 0.016 | $\hat{x}$ | estimated x | 1.793  |