8.1 Indefinite Integration is the reverse process of differentiation. *P.314*

If $\frac{d}{dx}[F(x)]=f(x)$, then the *indefinite integral* of ***f(x)*** is $\int f(x)dx=F(x)+C$, where ***C*** is called the constant of integration.

(a) Antiderivatives or Reversing the process of *Differentiation*

$$C'(x) = 100 \xrightarrow{\textbf{Integration}} \begin{array}{l} C(x) = 100x \\ C(x) = 100x + 500 \\ C(x) = 100x + 1000 \\ \vdots \\ \downarrow \\ C(x) = 100x + \boldsymbol{k} \end{array}$$

*where **k** is a constant*

(b) Notation of Antiderivative

$$\underbrace{\int}_{\text{Integral sign}} \underbrace{f'(x)}_{\text{integrand}} \underbrace{dx}_{\text{differential of } x} = f(x) + k$$

where k is constant of integration

8.2 Basic Integration Formulae *P.319*

(a)

Differentiation Rule	***Integration Rule***	
$\frac{d}{dx}(ax)=a$	$\int a\,dx = ax + c$	*Constant Rule*
$\frac{d}{dx}(x^n)=nx^{n-1}$	$\int x^n\,dx = \frac{x^{n+1}}{n+1}+c$	*Power Rule*
$\frac{d}{dx}(\ln x)=\frac{1}{x}$ *where* $x>0$	$\int \frac{1}{x}dx = \ln x + c$ *where* $x>0$	
$\frac{d}{dx}(e^{kx})=ke^{kx}$	$\int e^{kx}\,dx = \frac{1}{k}e^{x}+c$ *where* $k>0$	

(b) *Constant Multiple rule* $\int \mathrm{k} f(x)dx = \mathrm{k}\int f(x)dx$

Sum Rule $\int [f(x)+g(x)]dx = \int f(x)dx + \int g(x)dx$

Exercise 8.2 Q1, 3, 9, 17, 22, 28, 35, 38

8.3 Integration by Substitution *P.329*

Let ***u*** = ***g(x)*** and ***du*** = ***g'(x)dx***, then $\int f(x)dx = \int f(g(x))g(x)dx$

Exercise 8.3 Q1, 2, 7, 9, 11, 21, 26, 29, 31, 41, 45

EXERCISE 8.2

1. $\int 7\,dx = 7x + C$

3. $\int x^3\,dx = \frac{x^4}{4} + C$

9. $\int (x^2 + 4x - 3)\,dx = \frac{1}{3}x^3 + 2x^2 - 3x + C$

17. $\int \left(\frac{x-9}{x}\right)dx = \int \left(1 - \frac{9}{x}\right)dx$

$= x - 9\ln|x| + C$

22. $\int e^{-x}\,dx = -e^{-x} + C$

28. $\int \frac{3e^{6x} - e^{x}}{2e^{3x}}\,dx = \int \left(\frac{3}{2}e^{3x} - \frac{1}{2}e^{-2x}\right)dx$

$= \frac{3}{2}\left(\frac{1}{3}e^{3x}\right) - \frac{1}{2}\left(\frac{1}{-2}e^{-2x}\right) + C$

$= \frac{1}{2}e^{3x} + \frac{1}{4}e^{-2x} + C$

35. $\frac{dy}{dx} = 3x^2 + 2x - 1$

$y = \int (3x^2 + 2x - 1)\,dx$

$= x^3 + x^2 - x + C$

sub $(-1, 0)$ into y

$\therefore\ 0 = (-1)^3 + (-1)^2 - (-1) + C$

$\therefore\ C = -1$

$\therefore\ y = x^3 + x^2 - x - 1$

38. (a) $\frac{d^2y}{dx^2} = 6x - 12$

$\therefore\ \frac{dy}{dx} = \int (6x - 12)\,dx$

$= 3x^2 - 12x + C_1$

$y = \int (3x^2 - 12x + C_1)\,dx$

$\therefore\ y = x^3 - 6x^2 + C_1x + C_2$

sub $A(0, -8)$ & $B(3, 1)$ into y

$\therefore\ -8 = 0^3 - 6(0^2) + C_1(0) + C_2$

$C_2 = -8$

$1 = 3^3 - 6(3^2) + C_1(3) + C_2$

$1 = 27 - 54 + 3C_1 - 8$

$C_1 = 12.$

$\therefore\ y = x^3 - 6x^2 + 12x - 8$

(b) $\frac{dy}{dx} = 3x^2 - 12x + 12$

$= 3(x^2 - 4x + 4)$

$= 3(x - 2)^2$

$\therefore\ \frac{dy}{dx} \geq 0$ for all x

Thus, $y = f(x)$ is an increasing function.

EXERCISE 8.3

1. Let $u = 3x + 4$. Then $du = 3dx$.

$\int (3x + 4)^5\,du = \int u^5 \cdot \frac{1}{3}du$

$= \frac{1}{3}\left(\frac{1}{6}u^6\right) + C$

$= \frac{1}{18}(3x + 4)^6 + C$

2. Let $u = 1 - 2x$. Then $du = -2dx$

$\int (1 - 2x)^{27}dx = \int u^{27} \cdot \frac{-1}{2}du$

$= \frac{-1}{2}\left(\frac{1}{28}u^{28}\right) + C$

$= \frac{-1}{56}(1 - 2x)^{28} + C$

7. Let $u = 7x - 10$. Then $du = 7dx$

$\int e^{7x-10}\,dx = \int e^u \cdot \left(\frac{1}{7}du\right)$

$= \frac{1}{7}e^u + C$

$= \frac{1}{7}e^{7x-10} + C$

9. Let $u = 1 - 9x$. Then $du = -9dx$

$\int \frac{dx}{1 - 9x} = \int \frac{1}{u} \cdot \left(-\frac{1}{9}du\right)$

$= -\frac{1}{9}\ln|u| + C$

$= -\frac{1}{9}\ln|1 - 9x| + C$

11. Let $u = x^2 + 1$. Then $du = 2x\,dx$

$\int x(x^2 + 1)^5 dx = \int u^5 \cdot \frac{1}{2}du$

$= \frac{1}{2}\left(\frac{1}{6}u^6\right) + C$

$= \frac{1}{12}(x^2 + 1)^6 + C$

17. Let $u = x^3 - 1$. Then $du = 3x^2dx$

$\int x^2 e^{x^3-1}\,dx = \int e^u \cdot \frac{1}{3}du$

$= \frac{1}{3}e^u + C$

$= \frac{1}{3}e^{x^3-1} + C$

21. Let $u = e^x + 6$. Then $du = e^x\,dx$

$\int \frac{e^x}{e^x + 6}dx = \int \frac{1}{u} \cdot du$

$= \ln|u| + C$

$= \ln(e^x + 6) + C$

26. Let $u = 7 - 6\sqrt[3]{x^2}$. Then $du = -6 \cdot \frac{2}{3}x^{-\frac{1}{3}}dx$

$\int \frac{(7 - 6\sqrt[3]{x^2})^{-9}}{\sqrt[3]{x}} = \int u^{-9} \cdot \left(-\frac{1}{4}du\right)$

$= -\frac{1}{4}\left(\frac{1}{-8}u^{-8}\right) + C$

$= \frac{1}{32}(7 - 6\sqrt[3]{x^2})^{-8} + C$

29. $\int \frac{dx}{\sqrt{x}\,(3 + \sqrt{x})} = \int \frac{1}{3 + \sqrt{x}} \cdot 2d\,(3 + \sqrt{x})$

$= 2\ln|3 + \sqrt{x}| + C$

31. $\int \frac{\ln(3x)}{x}\,dx = \int \ln(3x)\,d(\ln(3x))$

$= \frac{1}{2}[\ln(3x)]^2 + C$

41. (a) $\frac{x}{2x-1} \equiv A + \frac{B}{2x-1}$

$\equiv \frac{A(2x-1)+B}{2x-1}$

$\therefore \quad x \equiv 2Ax + (-A + B)$

Equating coefficients,

$2A = 1$

and $-A + B = 0$

Hence, $A = \frac{1}{2}$ and $B = \frac{1}{2}$

(b) $\int \frac{x}{2x-1}dx = \int\left(\frac{1}{2}+\frac{\frac{1}{2}}{2x-1}\right)dx$ (from (a))

$= \frac{1}{2}x + \frac{1}{4}\int \frac{1}{2x-1}\,d(2x-1)$

$= \frac{1}{2}x + \frac{1}{4}\ln|2x-1| + C$

45. (a) $\frac{1}{x^2-1} \equiv \frac{A}{x-1} + \frac{B}{x+1}$

$\equiv \frac{A(x+1)+B(x-1)}{(x-1)(x+1)}$

$\therefore \quad 1 \equiv (A + B)x + (A - B)$

Equating coefficients,

$A + B = 0$

and $A - B = 1$

Hence, $A = \frac{1}{2}$ and $B = -\frac{1}{2}$

(b) $\int \frac{dx}{x^2-1}$

$= \int\left(\frac{\frac{1}{2}}{x-1} + \frac{-\frac{1}{2}}{x+1}\right)dx$

$= \frac{1}{2}\int \frac{1}{x-1}\,d(x-1) - \frac{1}{2}\int \frac{1}{x+1}\,d(x+1)$

$= \frac{1}{2}\ln|x-1| - \frac{1}{2}\ln|x+1| + C$

$= \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + C$