9.1 The **definite integral** of ***f(x)*** from a to b is denoted by $\int_a^b f(x)dx$. *P.354 - 355*

Thus $\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(c_i)\Delta x$ where $\Delta x = \frac{b-a}{n}$.

Evaluation of Definite Integral *P.356*

$$\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$$

Exercise 9.1 Q1, 5, 9, 13, 17

9.2 Properties of Definite Integral *P.357 - 362*

a. $\int_a^a f(x)dx = 0$

b. $\int_a^b f(x)dx = -\int_b^a f(x)dx$

c. $\int_a^b \mathrm{k} f(x)dx = \mathrm{k}\int_a^b f(x)dx$

d. $\int_a^b [f(x)+g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$

e. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

f. $\int_a^b f(x)dx = \int_a^b f(u)du$

Exercise 9.2 Q2, 4, 5

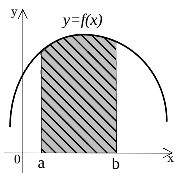
9.3 Substitution in Definite Integration *P.368*

If ***u* =g(x)** and ***du* = g'(x) dx**, then $\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$

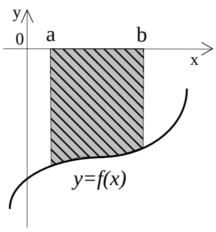
Exercise 9.3 Q1, 5, 7, 13, 17

9.4 Plane areas *P.373 - 377*

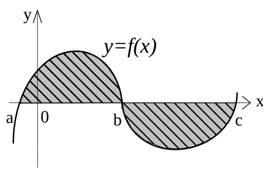
a. $\int_a^b f(x)dx$



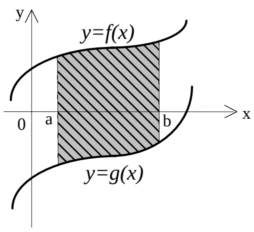
b. $\left|\int_a^b f(x)dx\right|$



c. $\int_a^b f(x)dx + \left|\int_b^c f(x)dx\right|$



d. $\int_a^b [f(x)\text{-}g(x)]dx$

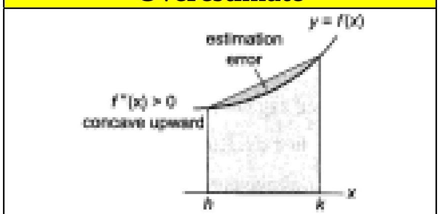
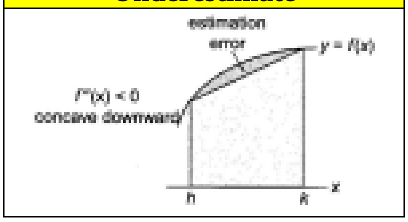


Exercise 9.4 Q8, 11, 13, 15 , 19, 20, 25, 26, 32, 37

9.5 Trapezoidal Rule *P.384 - 385*

a. $\int_a^b f(x)dx = \frac{\Delta x}{2}[f(x_0)+2f(x_1)+\ldots+2f(x_{n-1})+f(x_n)]$

b. *P.388*

Overestimate	**Underestimate**
	

Exercise 9.5 Q5, 7, 9, 16

EXERCISE 9.1

1. $\int_0^1 (3x+1)\,dx = \left[\frac{3x^2}{2}+x\right]_0^1 = \frac{3}{2}(1^2)+1 = \frac{5}{2}$

5. $\int_{-3}^{-1} (3x^2-4x+5)\,dx$

$= [x^3-2x^2+5x]_{-3}^{-1}$

$= [(-1)^3-2(-1)^2+5(-1)]-[(-3)^3-2(-3)^2+5(-3)]$

$= 52$

9. $\int_1^4 \frac{x^2-2x^3}{x^4}\,dx = \int_1^4 (x^{-2}-2x^{-1})\,dx$

$= [-x^{-1}-2\ln x]_1^4$

$= [-4^{-1}-2\ln 4]-[-1^{-1}-2\ln 1]$

$= \frac{3}{4}-2\ln 4$

13. $\int_{-3}^{0} (e^x+x^4)\,dx = \left[e^x+\frac{x^5}{5}\right]_{-3}^{0}$

$= \left(e^0+\frac{0^5}{5}\right)-\left[e^{-3}+\frac{(-3)^5}{5}\right]$

$= \frac{248}{5}-e^{-3}$

17. $\int_{-4}^{4} e^{-\frac{5x}{2}}\,dx = \left[-\frac{2}{5}e^{-\frac{5x}{2}}\right]_{-4}^{4}$

$= -\frac{2}{5}e^{-\frac{5(4)}{2}}-\left(-\frac{2}{5}e^{\frac{-5(-4)}{2}}\right)$

$= \frac{2}{5}(e^{10}-e^{-10})$

EXERCISE 9.2

2. (a) $\int_3^{-2} g(x)\,dx = -\int_{-2}^{3} g(x)\,dx = -(-4) = 4$

(b) $\int_2^3 -6g(x)\,dx = -6\int_2^3 g(x)\,dx = -6\times 5 = -30$

(c) $\int_{-2}^{2} g(x)\,dx = \int_{-2}^{3} g(x)\,dx + \int_3^2 g(x)\,dx$

$= -4-\int_2^3 g(x)\,dx$

$= -4-5 = -9$

4. (a) $\int_1^4 [f(x)+g(x)]\,dx$

$= \int_1^4 f(x)\,dx+\int_1^4 g(x)\,dx = 3+(-8) = -5$

(b) $\int_1^4 [2f(x)-5g(x)]\,dx$

$= 2\int_1^4 f(x)\,dx-5\int_1^4 g(x)\,dx = 2(3)-5(-8) = 46$

5. (a) $\int_{-5}^{-1} [3g(u)+4h(u)]\,du$

$= 3\int_{-5}^{-1} g(u)\,du+4\int_{-5}^{-1} h(u)\,du$

$= 3\int_{-5}^{-1} g(u)\,du-4\int_{-1}^{-5} h(u)\,du$

$= 3\times 8-4\times(-9) = 60$

5. (b) $\int_{-5}^{-5} [g(u)+h(u)]\,du = 0$

(∵ lower limit = upper limit)

(c) $\int_{-5}^{-1} [g(u)-h(u)+2]\,du$

$= \int_{-5}^{-1} g(u)\,du-\int_{-5}^{-1} h(u)\,du+\int_{-5}^{-1} 2\,du$

$= 8+\int_{-1}^{-5} h(u)\,du+[2u]_{-5}^{-1}$

$= 8+(-9)+[2(-1)-2(-5)] = 7$

EXERCISE 9.3

1. Let $u = 3x+1$, Then $du = 3dx$

When $x = 0$, $u = 3(0)+1 = 1$

When $x = 2$, $u = 3(2)+1 = 7$

$\therefore \int_0^2 (3x+1)^3\,dx = \int_1^7 u^3\cdot\frac{1}{3}\,du$

$= \left[\frac{1}{12}u^4\right]_1^7$

$= \frac{1}{12}(7^4-7^1) = 199.5$

5. Let $u = 5-x$, Then $dx = -dx$

When $x = 1$, $u = 5-1 = 4$

When $x = 4$, $u = 5-4 = 1$

$\therefore \int_1^4 \frac{1}{(5-x)^2}\,dx = \int_4^1 \frac{1}{u^2}\cdot(-du)$

$= \left[u^{-1}\right]_4^1$

$= 1^{-1}-4^{-1} = \frac{3}{4}$

7. Let $u = x^2+3$, Then $du = 2x\,dx$

When $x = 0$, $u = 0^2+3 = 3$

When $x = 1$, $u = 1^2+3 = 4$

$\therefore \int_0^1 x(x^2+3)^4\,dx = \int_3^4 u^4\cdot\frac{1}{2}\,du$

$= \left[\frac{1}{10}u^5\right]_3^4$

$= \frac{1}{10}(4^5-3^5) = 78.1$

13. Let $u = t^5+16t$, Then $du = (5t^4+16)\,dt$

When $t = 0$, $u = 0^5+16(0) = 0$

When $t = 2$, $u = 2^5+16(2) = 64$

$\therefore \int_0^2 (5t^4+16)(t^5+16t)^{\frac{1}{3}}\,dt = \int_0^{64} u^{\frac{1}{3}}\,du$

$= \left[\frac{3}{4}u^{\frac{4}{3}}\right]_0^{64}$

$= \frac{3}{4}(64^{\frac{4}{3}}-0^{\frac{4}{3}}) = 192$

17. $\int_0^3 x^2 e^{x^3}\,dx = \int_{x=0}^{x=3} e^{x^3}\cdot\frac{1}{3}\,d(x^3)$

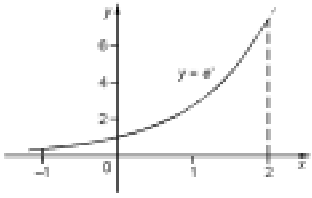
$= \left[\frac{1}{3}e^{x^3}\right]_0^3$

$= \frac{1}{3}(e^{3^3}-e^{0^3}) = \frac{1}{3}(e^{27}-1)$

EXERCISE 9.4

8. The required area $= \int_{-1}^{2} e^x \, dx$

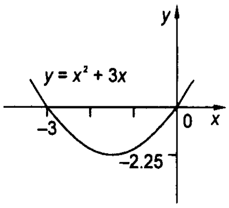
$= [e^x]_{-1}^{2} = e^2 - e^{-1}$



11. The required area $= -\int_{-3}^{0} (x^2 + 3x)\, dx$

$= -\left[\frac{x^3}{3} + \frac{3x^2}{2}\right]_{-3}^{0}$

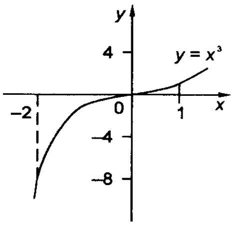
$= -\left[\frac{(-3)^3}{3} + \frac{3(-3)^2}{2}\right] = 4.5$



13. The required area $= -\int_{-2}^{0} x^3 \, dx + \int_{0}^{1} x^3 \, dx$

$= -\left[\frac{x^4}{4}\right]_{-2}^{0} + \left[\frac{x^4}{4}\right]_{0}^{1}$

$= \frac{(-2)^4}{4} + \frac{1^4}{4} = 4.25$

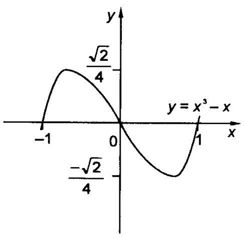


15. The required area $= \int_{-1}^{0} (x^3 - x)\, dx - \int_{0}^{1} (x^3 - x)\, dx$

$= \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^{0} - \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{0}^{1}$

$= -\left[\frac{(-1)^4}{4} - \frac{(-1)^2}{2}\right] - \left[\frac{1^4}{4} - \frac{1^2}{2}\right]$

$= \frac{1}{2}$



19. $y = \frac{2}{1-x}$

$\therefore \; x = 1 - \frac{2}{y}$

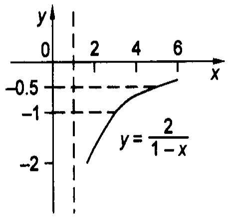
The required area

$= \int_{-1}^{-\frac{1}{2}} \left(1 - \frac{2}{y}\right) dy$

$= \left[y - 2\ln|y|\right]_{-1}^{-\frac{1}{2}}$

$= -\frac{1}{2} - 2\ln\frac{1}{2} - (-1 - 2\ln 1)$

$= \frac{1}{2} - 2\ln\frac{1}{2} = \frac{1}{2} + 2\ln 2$



20. $y = x^3$

$\therefore \; x = y^{\frac{1}{3}}$

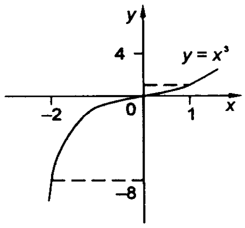
The required area

$= -\int_{-8}^{0} y^{\frac{1}{3}} \, dy + \int_{0}^{1} y^{\frac{1}{3}} \, dy$

$= -\left[\frac{3}{4} y^{\frac{4}{3}}\right]_{-8}^{0} + \left[\frac{3}{4} y^{\frac{4}{3}}\right]_{0}^{1}$

$= \frac{3}{4}(-8)^{\frac{4}{3}} + \frac{3}{4}(1)^{\frac{4}{3}}$

$= 12.75$



25. $y = \frac{1}{3}x$ ……(1)

$y = \sqrt{x}$ ……(2)

$\therefore \; \frac{1}{3}x = \sqrt{x}$

$\frac{1}{9}x^2 = x$

$\frac{1}{9}x(x-9) = 0$

$x = 0$ or 9 ……(3)

Putting (3) into (1),

when $x = 0$, $y = \frac{1}{3}(0) = 0$

when $x = 9$, $y = \frac{1}{3}(9) = 3$

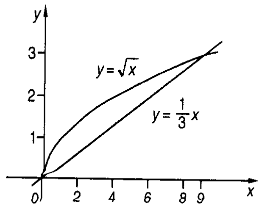
$\therefore$ The two curves intersect at (0, 0) and (9, 3)

25. The required area

$= \int_0^9 \left(\sqrt{x} - \frac{1}{3}x \right) dx$

$= \left[\frac{2}{3}x^{\frac{3}{2}} - \frac{1}{6}x^2 \right]_0^9$

$= \frac{2}{3}(9^{\frac{3}{2}}) - \frac{1}{6}(9^2) = 4.5$



26. $y = x$ ......(1)

$y = x^3$ ......(2)

Putting (1) into (2),

$x = x^3$

$x(x^2 - 1) = 0$

$\therefore$ $x = 0, \quad 1 \quad \text{or} \quad -1$ ......(3)

Putting (3) into (1),

$y = 0, 1$ or -1 respectively.

Thus, the points of intersection of (1) and (2) are (0, 0), (1, 1) and (–1, –1)

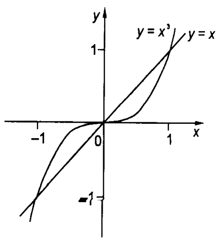
Due to symmetry,

the required area

$= 2\int_0^1 (x - x^3)\, dx$

$= 2\left[\frac{x^2}{2} - \frac{x^4}{4} \right]_0^1$

$= 2\left(\frac{1}{2} - \frac{1}{4} \right) = \frac{1}{2}$



32. (a) $y = 4x^2$

$\therefore \; x = \sqrt{\frac{y}{4}}$

$= \frac{1}{2}\sqrt{y}$

The area of the parabolic region

$= 2\int_0^{4a^2} \frac{1}{2}\sqrt{y}\, dy$

$= \left[\frac{2}{3}y^{\frac{3}{2}} \right]_0^{4a^2}$

$= \frac{2}{3}(4a^2)^{\frac{3}{2}}$

$= \frac{16}{3}a^3$

32. (b) Area of $\Delta AOB = \frac{1}{2} \times 2a \times 4a^2 = 4a^3$

$\therefore$ The required ratio $= 4a^3 : \frac{16}{3}a^3 = 3 : 4$

37. (a) C_1: $y = x^3$ ...... (1)

C_2: $y = \frac{1}{x}$ ...... (2)

$\therefore \; \frac{1}{x} = x^3$

$x^4 = 1$

$\therefore \; x = -1$ or 1 ...... (3)

Putting (3) into (2),

$y = -1$ or 1 respectively.

$\therefore$ The points of intersection are (–1, –1) and (1, 1)

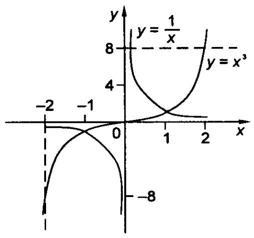
(b) The required area

$= \int_{-2}^{-1} \left(\frac{1}{x} - x^3 \right) dx$

$= \left[\ln|x| - \frac{x^4}{4} \right]_{-2}^{-1}$

$= \ln 1 - \frac{(-1)^4}{4} - \left[\ln 2 - \frac{(-2)^4}{4} \right]$

$= \frac{15}{4} - \ln 2$



(c) The required area

$= \int_1^8 \left(\sqrt[3]{y} - \frac{1}{y} \right) dy$

$= \left[\frac{3}{4}y^{\frac{4}{3}} - \ln|y| \right]_1^8$

$= \left[\frac{3}{4}(8^{\frac{4}{3}}) - \ln 8 \right] - \left[\frac{3}{4}(1^{\frac{4}{3}}) - \ln 1 \right] = \frac{45}{4} - \ln 8$

EXERCISE 9.5

5. $\Delta x = \frac{3-0}{2} = 1.5$

By trapezoidal rule,

$\therefore \; \int_0^3 x\, dx \approx \frac{1.5}{2}[0 + 2(1.5) + 3]$

$= 4.5$

The exact value $= \int_0^3 x\, dx$

$= \left[\frac{x^2}{2} \right]_0^3$

$= 4.5$

Note: When the graph of $y = f(x)$ is a straight line, the trapezoidal rule gives the exact value of the integral.

7. $\Delta x = \dfrac{-1-(-2)}{5} = 0.2$

By trapezoidal rule,

$$\int_{-2}^{-1} \frac{1}{x^2}\,dx$$

$$\approx \frac{0.2}{2}\left[\frac{1}{(-2)^2}+\frac{2}{(-1.8)^2}+\frac{2}{(-1.6)^2}+\frac{2}{(-1.4)^2}+\frac{2}{(-1.2)^2}+\frac{1}{(-1)^2}\right]$$

$= 0.505\ 8$ (4 sig. fig.).

The exact value $= \int_{-2}^{-1} \frac{1}{x^2}\,dx$

$= [-x^{-1}]_{-2}^{-1}$

$= -(-1)^{-1} + (-2)^{-1}$

$= 0.5$

9. $\Delta x = \dfrac{1-(-1)}{8} = 0.25$

By trapezoidal rule,

$$\int_{-1}^{1} e^x\,dx$$

$$\approx \frac{0.25}{2}(e^{-1} + 2e^{-0.75} + 2e^{-0.5} + 2e^{-0.25} + 2e^0 + 2e^{0.25} + 2e^{0.5} + 2e^{0.75} + e^1)$$

$= 2.363$ (4 sig. fig.).

The exact value $= \int_{-1}^{1} e^x\,dx$

$= [e^x]_{-1}^{1}$

$= e^1 - e^{-1}$

$= e - e^{-1}$

$= 2.350$ (4 sig. fig.)

16. By trapezoidal rule,
the surface area of the pool

$\approx \dfrac{5}{2}[0 + 2(12) + 2(13) + 2(14) + 2(15) + 2(17) + 8]$

$= 375\ \text{m}^2$.

The required amount of water $\approx 375 \times 1.8$

$= 675\ \text{m}^3$