12.1 Set Notation

(*Probability theory can be presented conveniently by using set notation*)

1. Set 集 / 集合 *(P.140)*

A set can be thought of as a collection of objects, called elements of the set.

	Set	Element
Notation	A, B, C	a, b, c
	or	
	E_1, E_2, E_3	a_1, a_2, a_3

Example E ＝ {x：x is a letter in the word 'cast'} or E ＝ { c, a, s, t }

2. Subset 子集 $\subset$ *(P.141-143)*

(a) If each element of a set A also belongs to a set B, we call A is a subset of B.
i.e. $A \subset B$, 'A is contained in B' or A is a subset of B
or $B \supset A$, 'B contains A'
E.g. If X ＝{YELLOW, RED, BLUE}, Y ＝{YELLOW, RED}, then $Y \subset X$.

(b) For all sets A, we have $A \subset A$.

(c) Equality of two sets
$A \subset B$ and $B \subset A \Leftrightarrow A = B$.
In such case, A and B have exactly the same elements.
Example 12.1

(d) Proper Subsets 真子集
$A \subset B$ but $A \neq B$.
E.g. {a, i, u} is a proper subset of {a, e, i, o, u}.
Example 12.2

(e) Universal set 宇集 **U**
A set contains all the objects.
E.g. If we toss a die, the set of all possible outcomes is the universe {1, 2, 3, 4, 5, 6}.
Example 12.3

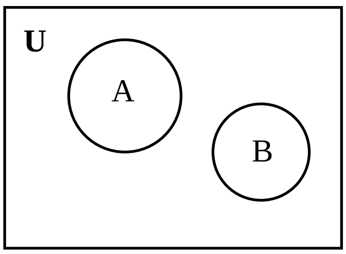
(f) Null set or Empty set 空集 $\varnothing$
A set having no element at all. It is a subset of any set.
E.g.{x：x is a positive integer and $x < 0$}

Example 12.4

Write down all the subsets of U＝{K, Q, J}.

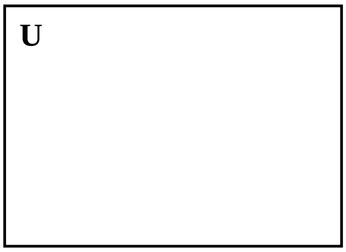
3. Venn Diagrams *(P.144)*

To represent a set by a simple plane area. (*Rectangle as Universal set and Circle as Sets or Subsets*). Sets of points inside the area represent elements of the set.



Example 12.5

Write down the relations between the sets U, X, Y, Z.

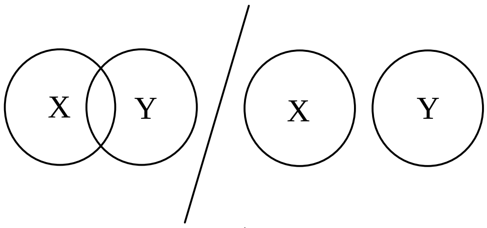


4. Operations運算 on Sets *(P.145-153)*

'∈' — if an element a belongs to a set E, we write $a \in$ E.

'∉' — if b does not belong to E, we write $b \notin$ E.

(a) Intersection 交集



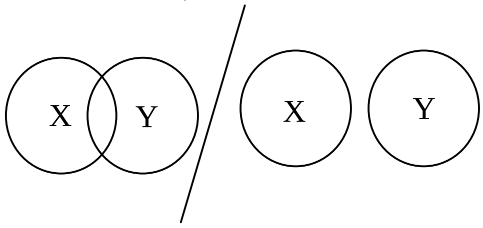
(i) X∩Y={x：x∈X and x∈Y}

(ii) X∩Y=∅ , X and Y are disjoint.

E.g. A={2, 4, 6, 8, 10}, B={3, 6, 9, 12}

A∩B=

(b) Union 併集



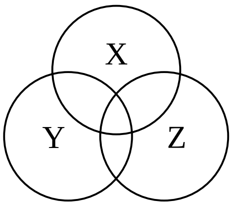
X∪Y={ x：x∈X or x∈Y }

E.g. A∪B=

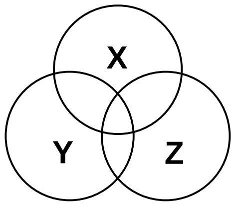
Example 12.6, 12.7, 12.8

(c) If more than 2 sets

X∩Y∩Z



X∪Y∪Z



E.g. A={x： $-1 \le x \le 1$}; B={x： $0 < x < 2$};

C={x： $0.5 < x \le 1.5$}.

A∩B∩C =

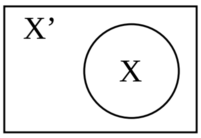
A∪B∪C =

Example 12.9, 12.10, 12.11

(d) Complement 餘集

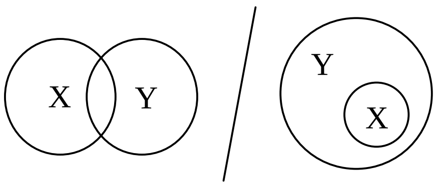
(i) Absolute complement

X'={x：x∈U and x∉X}



(ii) Relative complement

Y \ X={y：y∈Y and y∉X }



Example 12.12, 12.13, 12.14, 12.15

Exercise 12.1 Q16, 17

5. Some fundamental laws in set operations *(P.158)*

(a)	$X\cup X=X$	and	$X\cap X=X$	Idempotent Laws 冪等定律
(b)	$X\cup Y=Y\cup X$	and	$X\cap Y=Y\cap X$	Commutative Laws 交換律
(c)	$X\cup \varnothing=X$	and	$X\cap \varnothing=\varnothing$	Identity Laws 單位元律
	$X\cup U=U$	and	$X\cap U=X$	
(d)	$X\cup X'=U$	and	$X\cap X'=\varnothing$	Complementary Laws
	$(X')'=X$			

12.2 Sample Space & Events

1. Sample Space ↔ U *(P.163-166)*

   A set S which consists of all possible outcomes of a random (*outcome cannot be predicted with certainty*) experiment is called a sample space.

   E.g. 1. *Infinite sample space*

   If we toss a coin until a tail comes up, then

   $S=\{T, HT, HHT, \ldots\}$

   What is the tree diagram for S ?

   2. *Finite sample space*

   If two coins are tossed, then there are four possible outcome.

   $S=\{HT, HH, TT, TH\}$

   What is the tree diagram for S ?

   Example 12.16, 12.17, 12.18

2. Events *(P.168-171)*

   An event is a subset A of the sample space S.

   E.g. Two coins are tossed once. Let A be the event ‘at least one head occurs’ and B be the event ‘the second toss results in a tail’. Then

   $A=\{HT, TH, HH\}$

   $B=\{HT, TT\}$

   Example 12.20, 12.21

   (a) Impossible event 不可能事件

   No element in the sample space.

   (b) Certain event 必然事件

   An event contains the whole of the sample space.

   Example 12.22

   (c) Mutually exclusive events 互斥事件

   If two events A and B, such that $A\cap B=\varnothing$, then A, B are mutually exclusive. This means that they cannot both occur.

   (d) Complementary events 互補事件

   If $A\cap B=\varnothing$ and $A\cup B=S$, then A, B are complementary events. (*Two such events constituting the whole Sample space.*)

   (e) Exhaustive events 徹底事件

   The union of such events is equal to S. (*They may not be mutually exclusive to each other*)

   Example 12.23

12.3 Probability

1. Definition

   (a) Classical *(P.176)*

   If an event can occur in h different ways out of a total no. of n possible ways, all of which are equally likely or equiprobable, then the probability of the event is

$$P(A)=\frac{n(A)}{n(S)}=\frac{h}{n}$$

   E.g. Suppose we want the probability that a head turns up in a single toss of a coin. (Assume the coin is fair)

   A ={H} n(A) =

   S ={H, T} n(S) =

   ∴ P(head) =

   Example 12.24, 12.25, 12.26, 12.27, 12.28, 12.29

   (b) Frequency - empirical probability of an event / Relative Frequency *(P.181)*

   If after n repetitions of an experiment, where n is very large, an event is observed to occur in h of these, then the probability of the event is $\frac{h}{n}$.

   E.g. If we toss a coin 1000 times and find that it comes up heads 532 times, we estimate the probability of a head to be

   ____

   Example 12.30, 12.31

2. The Axioms of Probability *(P.184-185)*

   Since both classical and frequency approaches have serious difficulties, the first because the words 'equally likely' are vague 不明確 and the second because of the vagueness of the 'large no.' involved. Because of these difficulties, mathematicians have in recent years been led to an ***axiomatic*** 公理 approach to probability by using sets. *(P.187)*

   For every event A, in the sample space S,

   (a) $0 \leq P(A) \leq 1$

   (b) $P(S)=1$ Law of Certainty *P(certain event) = 1*

   (c) For any number of mutually exclusive events $A_1, A_2, \ldots$ in S,

   $P(A_1 \cup A_2 \cup A_3 \cup \ldots)=P(A_1)+P(A_2)+P(A_3)+\ldots$

   (d) $P(\varnothing)=0$ *P(impossible event) = 0*

   (e) $P(A')=1-P(A)$ Law for Complementary events

   Example 12.32, 12.33

Exercise 12.3 Q10, 16, 18, 20

12.4 Methods of Counting *(refer book 1/ ch.1)*

1. The multiplication Principle *Example 12.34, 12.35* *(P.191)*
2. Permutations & Combinations *Example 12.36, 12.37, 12.38, 12.39, 12.40* *(P.192-194)*
3. Applications to Probabilities *Example 12.41, 12.42, 12.43, 12.44* *(P.196)*

Exercise 12.4 Q8, 10, 17, 19, 25, 26, 28, 30; Revision Exercise 12 Q11

Self Studying

P.181 Weaknesses of the classical definition,

P.182 Weaknesses of the relative frequency definition.