13.1 The addition rule *(P.216-220)*

1. General addition rule
   (a) For *two* events
   $P(A\cup B)=P(A)+P(B)-P(A\cap B)$
   Example 13.1, 13.2
   (b) For *three* events
   $P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(A\cap C)+P(A\cap B\cap C)$
   Example 13.3
   (c) For *n* events
   $P(A_1\cup A_2\cup A_3\cup\ldots\cup A_n)=\Sigma P(A_k)-\Sigma P(A_j\cap A_k)+\Sigma P(A_i\cap A_j\cap A_k)-\ldots+(-1)^{n-1}P(A_1\cap A_2\cap\ldots\cap_n)$
   (d) **Law for the prob. Of Complementary Events**
   P(at least one of the events will occur)＝1－P(none of the events will occur)
   Example 13.4
   (e) For Mutually Exclusive events / Special Addition Rule
   $P(A_1\cup A_2\cup A_3\cup\ldots\cup A_n)=P(A_1)+P(A_2)+P(A_3)+\ldots+P(A_n)$
   Example 13.5

   Exercises 13.1 Q6, 10, 17

13.2 Conditional Probabilities *(P.224-225)*

Let A and B be two events, such that P(A) > 0, the probability of B given that A has occured：

$$P(B|A)=\frac{P(A\cap B)}{P(A)}$$

N.B. $P(B|A)\neq P(A|B)$
Example 13.6, 13.7, 13.8
Exercises 13.2 Q6, 11

13.3 The multiplication rule *(P.232-235)*

1. For *two* events
   $P(A\cap B)=P(A)P(B|A)$
   or $=P(B)P(A|B)$
   Example 13.9, 13.10
2. For *three* events
   $P(A\cap B\cap C)=P(A)P(B|A)P(C|A\cap B)$
   Example 13.11, 13.12

Exercises 13.3 Q5, 10

13.4 Independent events and the special multiplication rule *(P.242-243)*

For *k* events
$P(A_1\cap A_2\cap\ldots\cap A_k)=P(A_1)P(A_2)\ldots P(A_k)$
i.e. $P(B|A)=P(B)$,
the prob. of B occuring is not affected by the occurrence of A.
Example 13.13, 13.14, 13.15, 13.16
Exercises 13.4 Q3, 5

13.5 Bayes' Theorem

1. The total Probability Rule *(P.252-253)*

   If an event A must result in one of the mutually exclusive events $E_1, E_2, \ldots, E_n$, then

   $$P(A) = P(E_1)\,P(A \mid E_1) + P(E_2)\,P(A \mid E_2) + \ldots + P(E_n)\,P(A \mid E_n)$$

   Pf. : By Venn diagram P203 Fig.4.8, By tree diagram P204 Fig4.9.

   Example 13.17, 13.18

2. Bayes' Theorem (*Reversal of conditioning*) *(P.255-256, 259)*

   (a) For *two* events

   $$P(E|A) = \frac{P(E \cap A)}{P(A)} = \frac{P(E)P(A|E)}{P(E)P(A|E) + P(E')P(A|E')}$$

   (b) For *n* mutually exclusive and exhaustive events of sample space S, then
   In all the case of getting event A,

   $$P(E_i|A) = \frac{P(E_i)P(A|E_i)}{\sum_{i=1}^{n} P(E_i)P(A|E_i)} \qquad i = 1, 2, 3, 4, \ldots, n$$

   Example 13.19, 13.20, 13.21, 13.22

Exercises 13.5 Q4, 9

Self Studying

P261 Prior and posterior probabilities

EXERCISE (HKCEE Problems)

1. If A card is drawn at random from a pack of 52 playing cards, find the probability that
   (a) either a ' king ' or a ' queen ' is drawn,
   (b) either the ' king ' of hearts or the ' queen ' of clubs is drawn.
2. Two dice are thrown. Find the probability that
   (a) the sum of the numbers is greater than 9,
   (b) the sum of the numbers is not greater than 9.
3. A card is drawn at random from a pack of 52 playing cards. What is the probability of drawing an Ace or a Heart? (A pack of cards consists of four different suits, namely, Spades, Hearts, Diamonds, Clubs. Each suit has 13 cards, of which one and only one is an Ace). (1970)

4. In a throw of two dice, the following events are defined:
   E = Set of outcomes in which the two dice show the same number.
   F = Set of outcomes in which the sum of the numbers shown is 6.
   (a) What is the probability of
   i. E ii. F iii. E or F?
   (b) If the two dice are thrown twice, what is the probability that
   i. the two outcomes are in event E,
   ii. the first outcome is in E and the second outcome is in F? (1973)
5. If two dice are thrown once, find the probability that the sum of the numbers on the dice is
   (a) equal to 4, (b) less than 4, (c) greater than 4. (1982)