14.1 Random Variables (r.v.)

1. **Definition** *P.284 - 286*

   A r.v. is a *function* from a sample space S into real nos. It is usually denoted by a capital letter such as X or Y.

2. **Types** *P.288*

   Discrete r.v. & Continuous r.v.

3. **Example**

Experiment	Sample Space (S)	Random Variable (X)
Toss a fair coin twice	{ HH, HT, TH, TT }	X = total no. of heads { 0, 1, 2 }
Toss two dice	{ (i,j) i,j = 1, 2, 3, 4, 5, 6 }	X = sum of the numbers { 2, 3, . . . ,12 }

It should be noted that many other r.v. could be also be defined on this sample space, e.g. the square of the no. of heads, the no. of heads minus the no. of tails, etc.

Example 14.1, 14.2, 14.3

14.2 Probability distributions and probability functions *P.293 - 296*

Prob. function :

$P(X = x) = f(x)$ for $x \in A$ where A is the set of all possible values.

1. $0 \le f(x) \le 1$ for all $x \in A$.
2. $\sum_x f(x) = 1$

Prob. distribution also called prob. function, prob. Density, function for a discrete r.v.

Example 14.4, 14.5

Example

(a) Find the prob. function corresponding to the random variable X of tossing a fair coin twice.

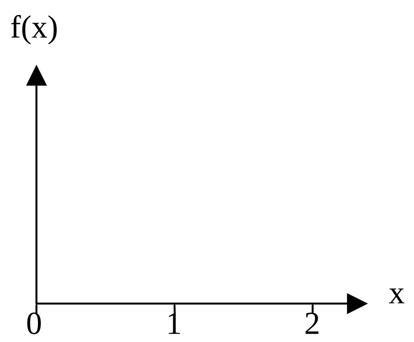
(b) Construct a prob. graph.

Soln

(a)

X			
f(x)			

(b)



14.3 Expectation (Mathematical expectation / expected value) *P.299 - 305*

For a r.v. X having the possible values $x_1, x_2, x_3, \ldots x_n$

$$E(x) = x_1P(X=x_1) + x_2P(X=x_2) + \ldots + x_nP(X=x_n)$$
$$= x_1f(x_1) + x_2f(x_2) + \ldots + x_nf(x_n)$$
$$= \sum_x x\,f(x)$$

Expectation of X is very often called the ***mean of X*** and is denoted by μ.

(The terms expectation, expected value, average value and the mean value are all used for the same concept. Like the mean value of a set of data, the expected value is used as a measure of central tendency of a pdf. While the mean is a measure of central tendency of an empirical distribution.)

Example 14.7, 14.8, 14.9

Example

1. A fair coin is tossed $10 will be awarded for a head and $5 will be charged for a tail. What is the expectation of gain.

   Soln

	H	T
gain		
Prob.		

E(gain) $=$

2. Suppose that a game is played with a single die assumed fair. In this game a player wins $20 if a 2 turns up, $40 if a 4 turns up, losses $30 if a 6 turns up, while he neither wins or losses if any other faces turn up. Find the expected sum of money won.

   Soln

   Let X be the sum of money won,

	1	2	3	4	5	6
X						
f(x)						

E(X) $=$

$=$

$\Rightarrow$ The player can expect to win .

$\Rightarrow$ In a fair game, therefore he should be expected to pay $5 in order to play the game.

Theorems on Expectation

1. If X is a r.v., and g(X) is also a r.v.

$$E[g(X)] = \sum_x f(x)g(x)$$

2. If X and Y are any r.v.,

$$E(X+Y) = E(X)+E(Y)$$

3. If c is any constant,

$$E(cX) = cE(X)$$

4. If X is a r.v., and a, b are any constants

$$E(aX+b) = aE(X)+b$$

5. If X is a r.v., and g(X), h(X) are also r.v.

$$E[g(X) \pm h(X)] = E[g(X)] \pm E[h(X)]$$

Example 14.8, 14.9, 14.10, 14.11

Exercise 14.3 Q4, 5

14.4 Variance & Standard deviation

P.308 - 314

$$\sigma^2 = E\left[(x-\mu)^2\right]$$

Proof：

Example 14.12, 14.13

Theorems on Variance

$$\sigma^2 = \sum_x f(x)(x-\mu)^2$$

1. $\sigma^2 = E(X^2) - \mu^2$ ＝

   Example 14.14, 14.15
2. If a is any constant,

   Var(a)＝0
3. If a is any constant,

   Var(aX)＝a^2Var(X)
4. If X is a r.v., and a, b are constants

   Var(aX＋b)＝a^2Var(X)
5. If X and Y are independent r.v.,

   Var(X ± Y)＝Var(X)＋Var(Y)

   Example 14.16

Exercise 14.4 Q2, 8; Revision Exercise 14 Q10

Standardized r.v. (Standard scores)－P.316 Q.17

Let X be a r.v. with μ and σ, then

$$Z = \frac{x-\mu}{\sigma}$$

properties of Z： E(Z) ＝ 0

Var(Z) ＝ 1

It is useful for comparing different distribution.