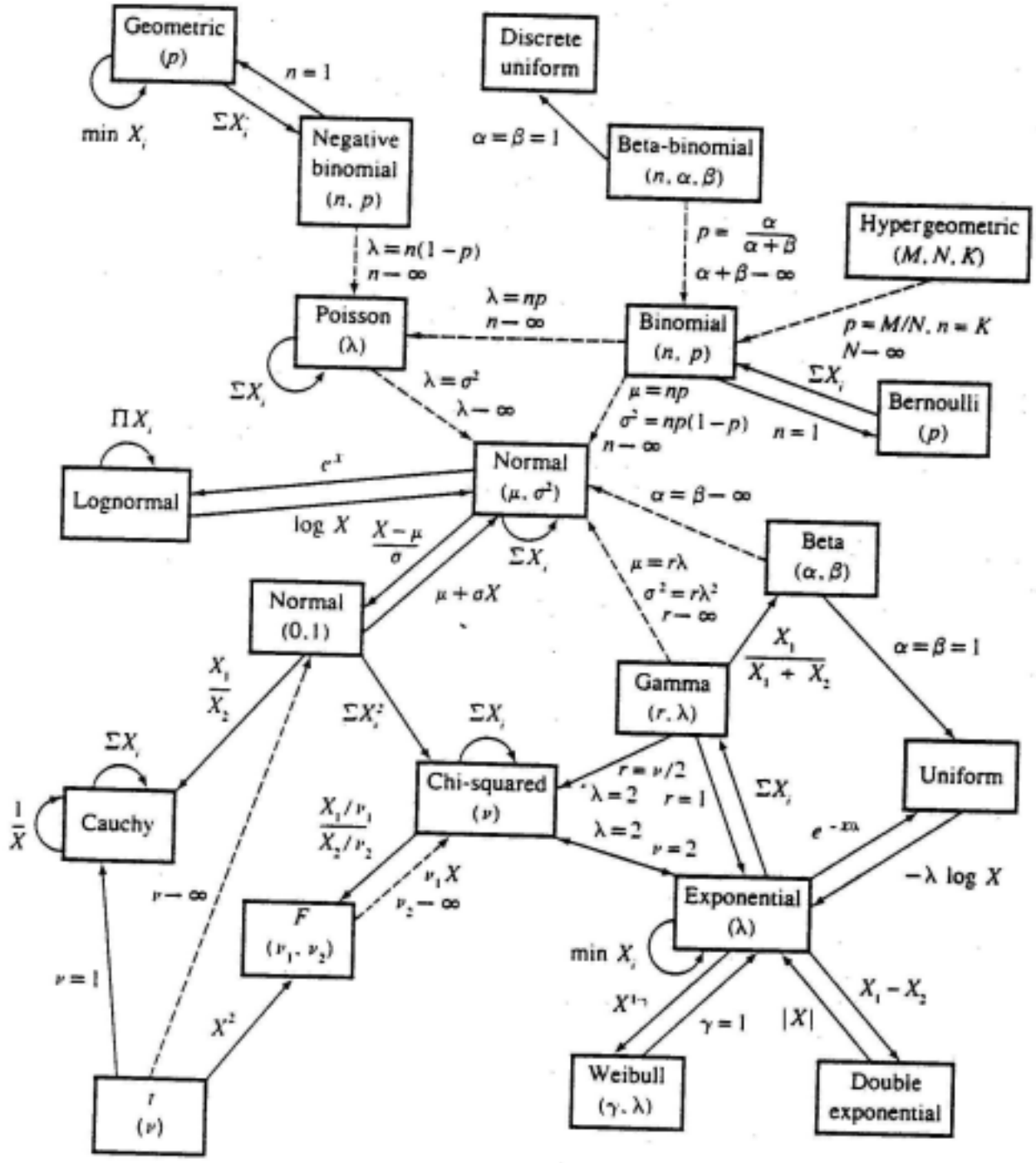
15.0 Common Distributions



Relationships among common distributions. Solid lines represent transformations and special cases, dashed lines represent limits. Adapted from Leemis (1986).

15.1 The Bernoulli Distribution (Bernoulli Trials) *P.330 - 331*

1. **Definition** Only 2 possible outcomes, e.g. sucess / failure, yes / no, good / defective, male / female, etc.

2. **Probability Function**

x	1	0
P (X = x)	p	1−p

$$\mathbf{P(X=x)} = p^x(1-p)^{1-x} \quad \text{for} \quad x = 1, 0.$$

3. **Mean & Variance**

mean μ =
= p

Variance σ^2 = **Var(X)** =
= $p(1-p)$

Example 15.1

Exercise 15.1 Q7

15.2 The Binomial Distribution *P.334 - 343*

1. **Definition** When n independent Bernoulli trials are carried out, the probability distribution becomes a Binomial Distribution.

2. **Probability Function**

Let X be the no. of successes in n trials, p be the prob. of success.
For n independent events,

P(x successes occur in n trials) = $C_x^n \underbrace{p \cdot p \cdot p \cdots p}_{x \text{ terms}} \cdot \underbrace{(1-p)(1-p)\cdots(1-p)}_{(n-x) \text{ terms}}$

$$\mathbf{P(X=x)} = C_x^n p^x (1-p)^{n-x} \qquad x = 0, 1, 2, \cdots, n.$$

3. **Notation** X ~ B(n, p) or X ~ b(x; n, p)

4. **Mean & Variance**

mean μ = np

Variance σ^2 = $np(1-p)$

Example 15.2
Exercise 15.2 Q1, 3, 8, 11

5. **Applications** Used in market research, public opinion survey, a company estimate its share of the market, etc.

Example 15.3, 15.4, 15.5, 15.6
Exercise 15.2 Q18, 19, 23

15.3 The Geometric Distribution *P.348 - 352*

1. **Definition** No. of trials (n) do not fix, but no. of successes is fixed (＝1)

   Example Find the prob. that in successive tosses of a fair die, a 3 comes up for the first time on the fifth toss.

   Required Prob. ＝

   ＝

2. **Probability Function**

   $$\mathbf{P(X = x) = (1-p)^{x-1} p}$$ for x = 1, 2, 3, ⋯. (*p*: prob. of success)

3. **Mean & Variance**

   mean μ $= \dfrac{1}{p}$

   Variance σ^2 $= \dfrac{1-p}{p^2}$

Example 15.7, 15.8, 15.9
Exercise 15.3 Q3, 6, 11, 15

15.4 The Poisson Distribution *P.355 - 365*

1. **Definition** It is the limiting form of binomial distribution as n→∞, p→0.

2. **Probability Function**

   $$\mathbf{P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}}$$ for x = 0, 1, 2,⋯.

3. **Notation** Po(λ)

4. **Mean & Variance**

   mean μ ＝ Variance σ^2 ＝ λ

   Example 15.10, 15.11

5. **Applications**
   (a) To describe the behaviour of rare events, e.g. serious floods, accidental release of radiation from a nuclear reactor *(Ex 15.4 Q14)*,
   (b) In insurance — count the no. of causualities 原因 *(Ex 15.4 Q8)*,
   (c) In quality control — count the no. of defects of an item *(Ex 15.4 Q24)*,
   (d) In biology — count the no. of bacteria in a specimen *(Ex 15.4 Q19)*,
   (e) In waiting time problems — count the no. of incoming telephone calls or incoming customers, no. of traffic arrivals *(Ex 15.4 Q25)*,
   e.g. trucks at terminals, airplanes at airport, ships at docks,
   (f) Extremely useful in determining the no. of typographical errors per page in a typed material *(Example 15.13)*, the no. of deaths as a result of road accidents *(Ex 15.4 Q13)* or the no. of deaths in a district in a specific period by a rare disease.

   Example 15.12, 15.13, 15.14, 15.15

Exercise 15.4 Q7, 13, 18, 19, 20, 23