16.0 Applications of Standard Deviation 常態分佈的應用 *(Certificate Level 中五程度)*

(1) Standard Score 標準分

The standard score z is given by

$$z = \frac{x - \bar{x}}{\sigma}$$

The standard score is often used for comparison purposes.

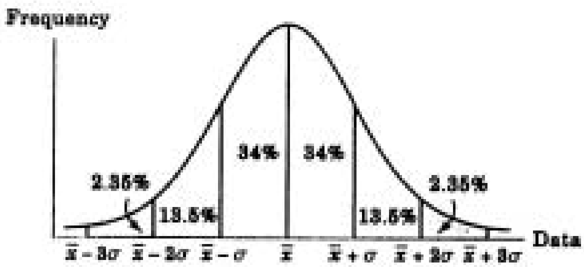
e.g.

	$\bar{x}$ *of the whole class*	σ *of the whole class*	*David's score*	*His standard score*
Mathematics test	74	4	76	$\frac{76-74}{4} = 0.5$
Physics test	53	8	65	$\frac{65-53}{8} = 1.5$

Since David's standard score in the Physics test is higher, his performance in the Physics test is better than that in the Mathematics test.

(2) Normal Distribution 常態分佈

The frequency curve of a normal distribution is bell-shaped and symmetrical about its mean.



For a normal distribution,

(i) about 68% of the data lie in the range $\bar{x} \pm \sigma$,

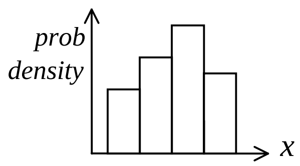
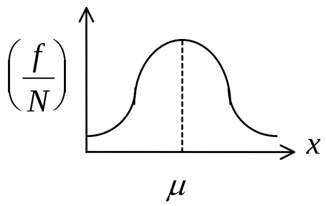
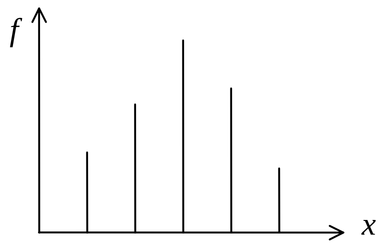
(ii) about 95% of the data lie in the range $\bar{x} \pm 2\sigma$,

(iii) about 99.7% of the data lie in the range $\bar{x} \pm 3\sigma$.

e.g. It is known that the marks of 200 students in a Mathematics test is normally distributed. If the mean is 63 and the standard deviation is 7, then the number of students scoring marks between 56 and 70 (i.e. between $\bar{x} - \sigma$ and $\bar{x} + \sigma$) is $200 \times 68\% = 136_{\#}$

16.3 The Normal Distribution

1. Comparison of Continuous & Discrete Distributions

Types	***Continuous prob. distribution***	***Discrete prob. distribution***
Examples	Normal Distribution (*Gaussian Distribution*)	Bernoulli Distribution, Binomial Distribution, Geometric Distribution, Poisson Distribution.
Graphical Representation	 *or*  *P.392*	
Expectation	$\mu = \int_{-\infty}^{\infty} x f(x)dx$ $\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$ *P.399*	$\mu = \sum x f(x)$ $\sigma^2 = \sum (x-\mu)^2 f(x)$ or $= E(X^2) - [E(X)]^2$
Conditions for prob. distribution	1. f(x) $\geq$ 0 2. $\int_{-\infty}^{\infty} f(x) = 1$ *P.396*	1. 1 $\geq$ f(x) $\geq$ 0 2. $\sum_{x} f(x) = 1$
Probability Distribution	*pdf* (prob density function) *P.405* $f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ *for* $-\infty < x < \infty$	*prob. function* f(x) $= p^x (1-p)^{1-x}$ for x = 1, 0. f(x) $= C_x^n p^x (1-p)^{n-x}$ for x = 0, 1, 2,$\cdots$, n. f(x) $= (1-p)^{x-1} p$ for x = 1, 2, 3, $\cdots$. f(x) $= \frac{\lambda^x e^{-\lambda}}{x!}$ for x = 0, 1, 2,$\cdots$.
Notation	N (μ , σ^2) *P.406*	B (n, p) / b (x; n, p), Geom(p) Po (λ)

2\. (a) Standard Normal Distribution $Z \sim N(0, 1)$ *P.407*

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}, \quad for\ -\infty < z < \infty$$

(b) Table of Standard normal prob. (Total area under the curve is equal to 1) *P.408*

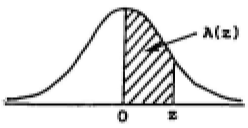
(c) How to read the table : *P.408*

(i) $z = 0.23$, Prob. $= 0.0910$ ⇒ P (0 ≤ Z ≤ 0.23) = 0.0910

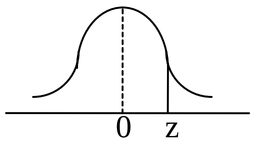
Table. Area under the Standard Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1712	.1808	.1844	.1879
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

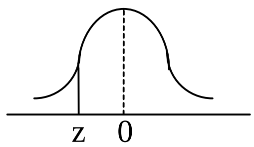
Note : An entry in the table represents the probability that a Standard Normal variable assumes a value between 0 and z. Probabilities for negative values of z are obtained by symmetry.



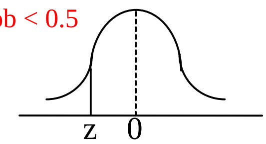
(ii) (I) P (0 ≤ Z ≤ z)



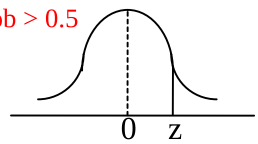
(II) P (z ≤ Z ≤ 0)



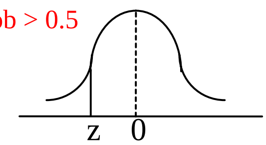
(III) P (Z ≤ z) (I) Prob < 0.5



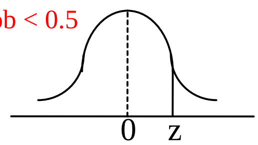
(II) Prob > 0.5



(IV) P (Z > z) (I) Prob > 0.5



(II) Prob < 0.5



Exercise 16.3 Q6, 7, 9

3\. Conversion of $N(\mu, \sigma^2)$ to $Z \sim N(0, 1)$ by *P.409*

$$z = \frac{x - \mu}{\sigma}$$

Exercise 16.3 Q12, 13, 16, 17

Remarks : *For continuous probability distribution* *P.395*

$P(h \le X \le k) = P(h < X \le k) = P(h \le X < k) = P(h < X < k)$

i.e. the prob is not affected by the inclusion or exclusion of the end points.

16.4 Applications of the Normal Distribution

Many distributions in daily life may be approximated by normal distribution.
Example 16.13

P.420

(a) ***First type*** Ex16.4 Q.1-5

3. Let T be the waiting time for a tram

$T \sim N(10, 3^2)$

(a) $P(T \le 5) =$
$=$
$=$
$=$
$=$

(b) $P(2 < T < 7) =$
$=$
$=$
$=$
$=$

(b) ***Second type*** Ex16.4 Q.6-13

12. Let X be the daily earnings

$X \sim N(500, 60^2)$

(a) $P(X > 600) =$
$=$
$=$
$=$
$=$

(b) $P(X > x) = 0.9$
$=$
$=$
$=$
$=$

(c) $P(X < Q_1) = 0.25$
$=$
$=$
$=$
$=$

$P(X < Q_3) = 0.75$
$=$
$=$
$=$
$=$

$\therefore$ IQR $=$

(c) ***Other type***

14. Let T be the delivery times

$T \sim N(30, 10^2)$

(a) $P(T > 35) =$
$=$
$=$
$=$
$=$

(b) $P(T > t) = 0.2$
$=$
$=$
$=$
$=$

$\therefore$ Expected cost $=$

Exercise 16.4 Q17, 20, 25